# Natural Frequency Analysis of Periodontal Conditions in Human Anterior Teeth

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Abstract-The purpose of this study was to evaluate the possibility of using natural frequency (NF) analysis to detect the attachment loss of periodontal tissue. In this study, 698 anterior teeth were examined by a conventional probing method and also by NF analysis. The teeth were triggered to vibrate with an impulse hammer, and the vibrational response was detected by an acoustic sensor. Our results demonstrate no significant difference in NF values between the upper-lower/left-right quadrants of the tested teeth, although the mean natural frequency value of central incisors with periodontal disease was found to be  $1.24\pm0.11$  kHz which is significantly lower than that of teeth in a healthy condition  $(1.34 \pm 0.20 \text{ kHz}; p < 0.01)$ . On the other hand, the mean frequency for periodontal disease involving canines (1.28±0.09 kHz) was also significantly lower than the corresponding value for healthy analogs  $(1.35\pm0.17 \text{ kHz})$ ; p < 0.05). These results suggest that NF analysis appears to be an effective method for assessing the periodontal condition of anterior teeth. Moreover, since this method is noninvasive, nondestructive, and necessitates minimal tooth contact, it can serve as an effective method for the early quantifiable testing and prevention of periodontal disease. © 2001 Biomedical Engineering Society. [DOI: 10.1114/1.1408925]

Keywords-Natural frequency, Tooth, Stability, Periodontium.

# **INTRODUCTION**

Early detection and diagnosis of altered periodontial conditions has always been a subject of concern for the dental researcher. Periodontal probe examination is the principle clinical method for the assessment of periodontal conditions *in vivo*, although the accuracy of this method is in doubt due to various factors such as variability in the diameter of the probe and the probing force applied possibly affecting the measurement results.<sup>2,8,17,20,21</sup> Radiographic image observation is another method for periodontal condition assessment. How-

ever, such a technique is not suitable for long-term toothmobility observation or the early diagnosis and prevention of periodontal disease. Therefore, there exists a strong clinical demand for a novel noninvasive technique to evaluate the status of periodontal conditions.

The natural frequency (NF) is an important dynamic response of a vibrating object, and it is related to the material properties and boundary conditions of a structure. The proposed technique for NF value measurement, modal testing, is a noninvasive and nondestructive method of investigation. The use of a NF value as a parameter for the assessment of bone density status and implant stability has been widely discussed in much published orthopedic research.<sup>3,4,6,11,12,25,26</sup> Recently, many *in* vivo and in vitro studies also demonstrated that a strong relationship exists between the NF value of a dental implant and the boundary conditions associated with that implant subsequent to implant installation.5,13,14,19,22 These studies have concluded that the NF value corresponding to selected dental implants could be used as an index for monitoring dental-implant stability and the degree of alveolar bone loss.

The application of such a vibration technique to human teeth is rare. In 1973, Noyes and Solt excited central incisors to vibrate by applying a sinusoidal force to the teeth. They found that the vibrational characteristics of a tooth are related to its mass and the nature of the tooth's surrounding tissue, although, the detailed relationship between NF and a tooth's boundary conditions was not discussed in their report.<sup>15</sup> Okazaki et al. studied the mobility of human teeth by using a modal testing technique in 1996, wherein the viscous damping behavior of the periodontal membrane and artificial material was compared and discussed in their study.<sup>16</sup> Although natural frequency analysis was not included in these authors' study, their results demonstrated that the modal testing technique could serve as a useful tool for analyzthe material properties of the periodontal ing

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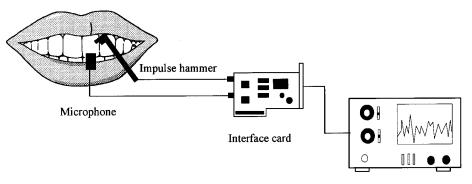


FIGURE 1. Instruments and testing methods used in this study.

Frequency spectrum analyzer

membrane.<sup>16</sup> To evaluate the correlation between tooth's NF values and its periodontium status, Lee *et al.* measured the natural frequencies of healthy upper central incisors *in vitro* and noted that the NF value was lower when the relative condition of surrounding structures was poorer.<sup>9</sup> Their findings suggest that NF values might be a useful parameter for assessing periodontal conditions clinically. To clearly understand the vibrational behavior of a tooth, the natural frequencies of anterior teeth were examined *in vivo* in this study, in which differences in NF values between these teeth with either healthy or diseased periodontium were compared and discussed.

## MATERIAL AND METHODS

To examine the changes in a tooth's NF value when a pathological condition arises in the periodontium locating that tooth, modal analysis and a periodontal-status examination, including a probing of the periodontal depth and x-ray radiation, were performed on 110 adult volunteers. Upper and lower central incisors and canines were chosen for examination for every volunteer. A total of 698 anterior teeth were tested in the entire study.

In the present study, the parameter "attachment loss" was used as an index for the assessment of the periodontal health condition, the term being defined as the sum of the probing depth and the extent of gingival recession. To examine the attachment loss, a Williams Probe (945188 pw Hu-Friedy, Chicago, IL) was used to probe six positions adjacent to the tooth being assessed including the mesiobuccal, midbuccal, distobuccal, mesiolingual, midlingual, and distolingual sides of the tested teeth.<sup>1</sup> According to Ramfjord's classification, an attachment loss reaching 4-6 mm was noted as moderate periodontal disease requiring treatment.<sup>18</sup> Therefore, a middle value of 5 mm was used as a threshold to judge the health status of the periodontal condition in this study. That is, if an attachment loss reaching 5 mm was detected in one of the six tooth-testing positions, then we gauged the tooth as exhibiting a diseased periodontium.

To measure the NF value of the tooth, the modal testing method was adopted in this study. As shown in Fig. 1, an impulse force was applied directly to the labial surface of the tested tooth with an impulse force hammer (model GK291C80, PCB Piezotronics, Buffalo, NY). When the piezoelectronic tip of the hammer touched the tooth, an impact signal was transferred to the analyzer. According to our previous study,<sup>7</sup> in order to minimize the mass effect of the sensor, a noncontact acoustic microphone (FM-10B, FC Electronic, Taipei, Taiwan) was placed at a distance of about 1 cm away from the labial surface of the tested tooth to receive the vibrational signal.

To diminish artifacts and any spurious responses caused by background noise, the microphone was not turned on until the analyzer had received the hammerstriking signal. The vibrational signal was then transferred to a frequency spectrum analyzer (Probel II, Prowave Engineering, Hsinchu, Taiwan). The specific natural frequency of the tested tooth was determined as being that frequency corresponding to the peak vibrational amplitude value of the tooth. Each datum point was obtained by averaging the frequencies of five repeated measurements.

Statistical software (Stastica, Statosoft, Tulsa, OK) was used for comparisons of measured NF values and the variables corresponding to the tested teeth. The tested teeth were divided into four groups according to their position in the four intraoral quadrants as upper left (UL), upper right (UR), lower left (LL), and lower right (LR). One-way analysis of variance was computed to test if the quadrant location of the tooth was related to the value of its natural frequency for both healthy and diseased groups of teeth. On the other hand, comparisons of NF values with healthy and diseased periodontium were made with the use of the Student's t-test with p values below 0.05 being considered to indicate a statistically significant difference between the means of test populations.

TABLE 1. Number of examined teeth distributed by the tooth group and general periodontal health condition. UL, upper left; UR, upper right; LL, lower left; and LR, lower right.

	Healthy			Diseased				
Tooth	UL	UR	LL	LR	UL	UR	LL	LR
Incisors Canines	56 57	80 81	77 81	44 47	6 5	8 7	15 11	13 10

#### RESULTS

In total, 698 anterior teeth were tested in this study. After an attachment-loss examination was performed, we determined that there were 523 teeth with healthy periodontium (including 257 central incisors and 266 canines) and 75 teeth exhibiting periodontal disease (including 42 central incisors and 33 canines). The distributions by health status of the tested teeth are listed in Table 1.

Figure 2 illustrates a typical signal of a tested incisor received by an acoustic microphone. The amplitude of the vibrational spectrum decayed in 10 ms due to the damping effect of the surrounding tissue. While the vibrational signal was converted from the time domain to the frequency domain, the natural frequency of the test incisor could be clearly identified through the peak value of its mobility (Fig. 3).

The NF values corresponding to tested teeth at different locations within the oral cavity with healthy and diseased periodontia are listed in Tables 2 and 3, respectively. When the NF values of tested teeth are grouped according to their location in different sites of the four quadrants, regardless of whether a tooth is healthy or diseased, no significant difference in NF was noted between the various quadrants. According to these findings, the tested teeth in the various quadrants were merged in the following analysis.

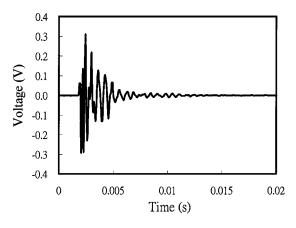


FIGURE 2. Vibrational signal of a tested incisor as received by the acoustic microphone.

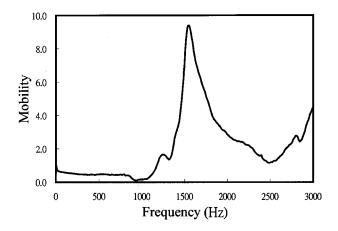


FIGURE 3. Frequency response spectrum of a vibrating incisor. A specific tooth's natural frequency was identified by the mean of the resonance frequency corresponding to the peak value of the vibration amplitude for replicate experiments.

Table 4 lists the mean NF values for incisors and canines after combining all the quadrant groups. The mean NF values for both central incisors and canines varied quite obviously as a result of the occurrence of periodontal disease. The measured natural frequencies of healthy central incisors (with an average of 1.34  $\pm 0.20$  kHz) and the diseased group (with an average of  $1.24 \pm 0.11$  kHz) ranged from 0.93 to 1.82 and 1.02 to 1.54 kHz, respectively. The Result obtained from the statistics showed that the NF value of healthy incisors differed substantially from the corresponding vibration frequency for analogous teeth demonstrating periodontal disease (p < 0.01). In addition, the same tendency was also noted for the canine groups. The measured natural frequencies of healthy canines (with an average of 1.35  $\pm 0.17$  kHz) and the diseased group (with an average of  $1.28\pm0.09$  kHz) ranged from 0.96 to 1.82 and 1.11 to 1.51 kHz, respectively. The mean frequency for healthy canines (Hz) also differed notably from the corresponding value for analogous teeth demonstrating periodontal disease (p < 0.05). According to traditional classifications, central incisors and canines were combined as anterior teeth to test the natural frequency changes with the occurrence of periodontal disease. The mean frequency for the periodontal involved grouped anterior teeth  $(1.26\pm0.10 \text{ kHz})$  was significantly lower than the corresponding value for analogous teeth revealing healthy periodontia  $(1.34 \pm 0.18 \text{ kHz}; p < 0.01)$ .

#### DISCUSSION

An accelerometer is the most common transducer used for detecting vibrational signals in modal testing experiments. Both Noyes and Solt<sup>15</sup> and Okazaki *et al.*<sup>16</sup> used an accelerometer as a signal detector in their vibra-

TABLE 2. Natural frequencies of healthy teeth in different locations (mean  $\pm$ SD, kHz) within the oral cavity. No significant difference in NF was noted between the various quadrants.

Tooth	UL	UR	LL	LR	<i>p</i> value
Incisors	1.34±0.19	1.35±0.20	$1.34 \pm 0.20$	1.32±0.020	0.9304
Canines	1.32±0.17	1.35±0.18	$1.38 \pm 0.16$	1.32±0.18	0.1165

tional analyses of teeth. However, due to the relatively small size of human teeth by comparison to the accelerometer, using an accelerometer as a detector may cause unexpected errors. On the other hand, when an accelerometer is used, it needs to be firmly attached to the surface of the test object. Therefore, the use of an accelerometer as a signal detector for the vibrational analysis of tested teeth is not suitable for intraoral practices. To eliminate the unnecessary mass effect of the accelerometer, Lowet et al. used a microphone as a transducer to measure the natural frequency of the tibia. These authors discussed how using a microphone as a sensor reflects certain advantages in examining the vibrational behavior of hard tissue of living creatures.<sup>11</sup> In our study, a noncontact acoustic microphone was used to collect the vibrational responses of tested teeth. Our results indicated that a microphone can also serve as an effective tool for the testing of NF values for human teeth due to its absence of contact with the tooth being tested and, if selected appropriately, its highly sensitive characteristics.

As indicated by the data listed in Tables 2 and 3, no difference was found between the NF values for teeth on the basis of the quadrant in which they were located. This would likely be the case since the normal human dentition reveals a generally equivalent symmetrical geometric shape and distribution between the four quadrants inside the oral cavity. Although the results of Noves and Solt indicated that an observable difference of natural frequencies of teeth existed between the lower and upper central incisors,<sup>15</sup> this observation may be questionable due to the small sampling size adopted in their analysis. The results of the Okazaki et al. analysis revealed that incisors exhibit similar vibrational characteristics regardless of whether the teeth were located on the left side or the right side.<sup>16</sup> These results are similar to our findings. The effect of this finding shows that when periodontal disease occurs, we can use a person's contralateral analogous healthy tooth as a basis for a reference value to test the level of periodontal attachment for corresponding teeth.

In our previous study, a finite-element model was used to study the vibrational characteristics of maxillary central incisors. The first four modes the of NF values and the corresponding mode shapes were calculated. We found that the first vibration mode of the model is a single bending mode.<sup>9</sup> According to the above finding and considering the mechanical structure of a tooth (free on one end, and fixed on the other), thus, with regards to its vibrational behavior, we can analyze a tooth's natural frequency by using a cantilever beam theory as follows:<sup>11,13,24</sup>

$$f_n = \alpha \sqrt{\frac{EI}{\rho \iota^4}}.$$
 (1)

Equation (1) expresses the natural frequency of a vibrating cantilever beam, where  $f_n$  is the natural frequency of the beam,  $\iota$  is the effective length of its oscillation, E is Young's modulus, I is the moments of inertia,  $\rho$  is the mass per unit effective length, and  $\alpha$  is the relevant constant of the boundary term. According to the previous classification, the alveolar bone can be divided into four different types.<sup>10</sup> Different alveolar bone types will thus provide different boundary conditions for teeth, and for this reason, teeth with these different bone types will exhibit different values of  $\alpha$ . Although the exact effect of surrounding tissue on the value of  $\alpha$  is still unknown, however, in the presence of the same alveolar bone type, the value of  $\alpha$  can be assumed to be a fixed value. In this study, x-ray images of the tested teeth were examined for bone quality. All the tested teeth selected in this study were confirmed to not show bone density decreases. Because the structural properties affecting the natural frequency, i.e., Young's modulus, density, and the shape of the cross section, should not vary in a wide range be-

TABLE 3. Natural frequencies of periodontal disease involving teeth in different locations (mean  $\pm$ SD, kHz) within the oral cavity. No significant difference was noted between the various quadrants.

Tooth	UL	UR	LL	LR	<i>p</i> value
Incisors Canines	1.25±0.07 1.32±0.15	$\begin{array}{c} 1.24 {\pm} 0.12 \\ 1.29 {\pm} 0.10 \end{array}$	$\begin{array}{c} 1.27 {\pm} 0.11 \\ 1.26 {\pm} 0.07 \end{array}$	$\begin{array}{c} 1.22 {\pm} 0.12 \\ 1.27 {\pm} 0.08 \end{array}$	0.6554 0.6203

TABLE 4. Comparison of natural frequency (mean  $\pm$  SD, kHz) between healthy and periodontal disease involving anterior and posterior teeth.

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Tooth	Healthy	Diseased	<i>p</i> value	
Incisors	$1.34 \pm 0.20$	$1.24 \pm 0.11$	<0.01	
Canines	$1.35 \pm 0.17$	$1.28 \pm 0.09$	< 0.05	
Anterior teeth	$1.34 {\pm} 0.18$	$1.26 \pm 0.10$	<0.01	

tween individuals. Therefore, the natural frequencies of the teeth will only be affected by the effective length of vibration ( $\iota$ ). On the other hand, because the material properties of tested teeth, such as Young's modulus and tooth density, are not likely to change despite the presence of periodontal disease, it would appear that the main factors that influence the NF values derive primarily from the tooth's boundary factors and the value of the effective vibrational length of the tooth. In this study, we found that the natural frequency of both central incisors and canines was lower when the teeth had an attachment level loss larger than 4 mm.

The extent of the attachment level loss is essentially the decrease in the contact surface between the root and the surrounding tissue, thus, a decrease in the height of the periodontium results in an increase of exposed tooth height, i.e., the effective vibration height. Such a result enhances the ease with which the tooth may vibrate when stimulated appropriately. Therefore, according to Eq. (1), the NF values of a tooth should decrease when the tooth's attachment level is reduced. As shown in Table 4, our results demonstrate a significant tendency for the NF value of a tooth to decrease subsequent to the occurrence of periodontal attachment loss. This finding matches the vibration theories well. However, for each type of tooth studied, the mean of the diseased data set was within one standard deviation of the mean for the normal data set. Also, the mean for the normal data set was within one standard deviation of the mean for the diseased data set. This may be because attachment loss reaching 5 mm in one of the six tooth-testing positions was detected. We then assess the tooth as one having a diseased periodontium. In fact, mechanically, a 5 mm loss in a single location probably does not influence the natural frequency as much as a 4 mm loss in a number of locations. To better understand the effect of surrounding tissue on the vibrational characteristics, the NF values of the tested teeth were plotted versus the sum of the fixation loss for both types of teeth. These data are demonstrated in Fig. 4. Regardless of the type of teeth, the results of the regression analysis indicate a statistically significant negative linear relationship between the NF value and the sum of the attachment loss.

Referring to the previous observation, the contact surface of the posterior tooth is almost twice as large as that of the anterior teeth.<sup>23</sup> Since the root numbers and shapes

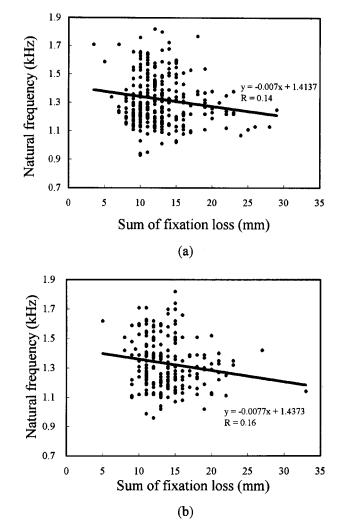


FIGURE 4. Relationships between NF value and the sum of fixation loss in tested incisor (a) and canine (b). Scatter plots show the relationship between NF value and sum of fixation loss. Solid lines represent the best-fit linear regression.

of the anterior and posterior teeth are completely different, their dynamic behaviors under induced vibration will also be different.<sup>16</sup> However, the vibrational characteristics of posterior teeth were not assessed in this study because the use of an impulse force hammer in this study was not a simple procedure for posterior teeth examination.<sup>16</sup> For the purposes of developing a new clinical diagnostic method for detecting the degree of periodontal health, a novel force-triggering device for individual teeth and suitable for intraoral performance is undoubtedly needed.

In conclusion, we find that the mean natural frequency of anterior teeth will indeed significantly decrease in value as a result of tooth periodontium attachment loss greater than 4 mm. The findings of this research show that the NF analysis for an anterior tooth has the potential to be regarded as a novel noninvasive, nondestructive, minimum-contact, and instant method for clinical periodontal condition assessment.

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